Mass Transfer to Newtonian and Non-Newtonian Fluids in Short Annuli

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INTRODUCTION

Mass transfer to fluids flowing through annuli is frequently encountered in many industrial processes. Corrosion, scaling and descaling, annular condensers, transpiration and film cooling of annular ducts, and annular electrochemical reactors are some typical examples of the practical application of mass transfer in annular systems. Cases of mass transfer in fully developed laminar and turbulent annular flows with both developed and developing concentration boundary layers have been adequately dealt with by several workers (Ross and Wragg, 1965; Wragg and Ross, 1967, 1968; Turitto, 1975; Pickett, 1977; Remorino et al., 1979). Cases of developing annular flows with developing concentration boundary layers have received little attention (Pickett, 1977). Further, except for the work of Remorino et al. (1979), almost all reported studies of mass transfer in annuli are limited to Newtonian fluids.

The present paper reports new experimental data on mass transfer to non-Newtonian fluids and shows that Newtonian relations could easily be extended to such fluids. New design equations for both developing laminar and turbulent flows in short annuli are also presented.

BACKGROUND

Newtonian Fluids

Ross and Wragg (1965) have made the most comprehensive theoretical and experimental contributions regarding the study of mass transfer in fully developed laminar and turbulent flows in an annulus. For fully developed laminar flow, extension of the Leveque solution to an annulus gave

$$Sh = 1.614[\phi(a) \cdot Re \cdot Sc(d_e/1)]^{1/3} \tag{1}$$

where $\phi(a)$ is a function of aspect ratio $a(=D_i/D_o)$ and is given by

$$\phi(a) = \frac{(1-a)}{a}\left[\frac{0.5 - \dfrac{a^2}{(1-a^2)}\ln\left(\dfrac{1}{a}\right)}{\dfrac{(1+a^2)}{(1-a^2)}\ln\left(\dfrac{1}{a}\right) - 1}\right] \tag{2}$$

For fully developed turbulent flow with $L/d_e < 2$, Ross and Wragg (1965) showed that their data agree well with the equation

$$Sh = 0.276\, Re^{0.58} \cdot Sc^{1/3}(d_e/L)^{1/3} \tag{3}$$

developed analytically for very short parallel plates in a fully developed flow (Pickett, 1977). A more recent analysis by Pickett (1977), however, indicates that

$$Sh = 0.145\, Re^{2/3} \cdot Sc^{1/3}(d_e/L)^{1/3} \tag{4}$$

gives a better agreement with the experimental data. For $L/d_e >$ 2, Lin et al. (1951) obtained good agreement with the Chilton-Colburn equation

$$Sh = 0.023\, Re^{0.8} \cdot Sc^{1/3} \tag{5}$$

A theoretical analysis of the mass transfer problem for an annulus with short transfer surfaces and having no hydrodynamic entrance length is difficult to make. The only reported experimental data on mass transfer from short electrodes in developing laminar flow are those of Carbin and Gabe (1974), who correlated their results by

$$Sh = 3.93\, Re^{0.32} \cdot Sc^{0.33}(d_e/L)^{0.35} \tag{6}$$

Considering the similarity between Eqs. 1 and 6, Pickett (1977) recommended the use of Eq. 1 for design purposes, ignoring the hydrodynamic entrance effects, if any.

Non-Newtonian Fluids

For fully developed laminar annular flow of a non-Newtonian fluid obeying the power law model

$$\tau = K\left(\frac{du}{dr}\right)^n \tag{7}$$

Equation 1 can be written as (Remorino et al., 1979)

$$Sh = 1.614\left\{Re_p \cdot Sc_p\left(\frac{d_e}{L}\right) \cdot \Gamma(a,n)\right\}^{1/3} \tag{8}$$

where Re_p and Sc_p are modified Reynolds and Schmidt numbers, respectively, for power law fluids and

$$\Gamma(a,n) = \frac{(1-a^2)a^{1/2}\left\{\left[\dfrac{\lambda(a,n)}{a}\right]^2 - 1\right\}^{1/n}(1-a)}{4\Omega(a,n)} \tag{9}$$

Fredrickson and Bird (1958) have reported the numerical values of $\lambda(a,n)$ and $\Omega(a,n)$. Remorino et al. (1979) have shown that experimental data for both Newtonian and non-Newtonian fluids (n = 0.77 to 1.0) can be successfully represented by Eq. 8.

TABLE 1. RANGE OF OPERATING VARIABLES

Variable	Source: Present Work	Carbin & Gabe (1974)	Singh *et al.* (1971)
Technique	Dissolution	Electrochemical	Dissolution
System	Benzoic acid/aq./CMC solution	$CuSO_4$–H_2SO_4 solution	Benzoic acid-water
Outer cylinder dia., D_o, cm	10.00	3.17	4.5
Inner cylinder dia., D_i, cm	2.0–2.65	0.632	1-6–2.25
Aspect ratio, a	0.2–0.265	0.199	0.355–0.5
d_e/L	1.75–3.125	2.307	0.564–0.725
Fluid	1% aq. CMC solution	Aq. solution of H_2SO_4 and $CuSO_4$	Water
	$n = 0.9365$ $K = 1.12$ $g{\cdot}s^{2-n}/cm$	$n = 1$, $\nu = (1.005–1.025) \times 10^{-2}$ cm^2/s	$n = 1$ $\mu = 0.81–0.886$ cp
D, cm^2/s	9.47×10^{-6}	$(6.1–6.39) \times 10^{-6}$	$(9.67–10.00) \times 10^{-6}$
Reynolds no.	0.023–26.2	8.24–327.3	150–19,000
Schmidt no.	$(1.084–1.652) \times 10^5$	1,572–1,680	800–1,100

EXPERIMENTAL

During the course of the mass transfer investigations from cylinders in cross flow of non-Newtonian fluids (Ghosh et al., 1984) a few sets of measurements were also made with cylinders oriented parallel to the flow. Mass transfer coefficients were determined by the dissolution method, measuring the weight losses of benzoic acid cylinders exposed to 1% sodium-carboxymethyl cellulose (Na-CMC) solution. The experimental setup was essentially similar to that used earlier (Kumar et al., 1980; Kumar and Upadhyay, 1981). In all cases accurately weighed test specimens, with their ends blinded with wax, were coaxially mounted in a vertical glass column of 10 cm dia. The system thus formed a short annulus with developing concentration and hydrodynamic boundary layers. The rheological characteristics and other physicochemical parameters for Na-CMC solutions were obtained experimentally as described elsewhere (Lal et al., 1980; Kumar and Upadhyay, 1981). The range of various parameters covered in the present work are given in Table 1.

RESULTS AND DISCUSSION

Laminar Regime

Wragg and Ross (1967) extended the Martinelli and Boelter analysis for heat transfer to an annular geometry and showed that

$$Sh = 1.96\{Re \cdot Sc(d_e/L) + 0.04[Gr \cdot Sc(d_e/L)]^{0.75}\}^{1/3} \quad (10)$$

expresses the Sherwood number as a function of dimensionless groups describing the contributions of both free and forced convection for fully developed laminar flow. A plot of Sh against $Re \cdot Sc(d_e/L)$ with $Gr \cdot Sc(d_e/L)$ as parameter produces a family of curves as shown in Figure 1. The similarity between Figure 1 and Figure 2, in which present results are plotted along with the data of Carbin and Gabe (1974), indicates that a relation similar to Eq. 9 should be valid for developing boundary layers also.

From Eq. (10) it is clear that a plot of Sh vs. $[\phi(a)]\{Re \cdot Sc(d_e/L) + 0.04[Gr \cdot Sc(d_e/L)]^{0.75}\}$ on log-log coordinates should result in a straight line. Such a plot, as shown in Figure 3, displays a good straight-line relationship, which can be represented by

$$Sh = 2.703\Gamma(a,n)^{1/3}\{Re_p \cdot Sc_p(d_e/L) + 0.04[Gr_p \cdot Sc_p(d_e/L)]^{0.75}\}^{1/3} \quad (11)$$

This equation provides a good correlation of the non-Newtonian data and compares well with Eq. 10 proposed by Wragg and Ross (1967). The higher coefficient of Eq. 11 compared to that of Eq. 10 is due to the developing nature of the hydrodynamic and concentration boundary layers. At $(Gr_p \cdot Sc_p \cdot d_e/L) = 0$, Eq. 11 reduces to

$$Sh = 2.703[Re_p \cdot Sc_p \cdot \Gamma(a,n)(d_e/L)]^{1/3} \text{ (for } n < 1)$$

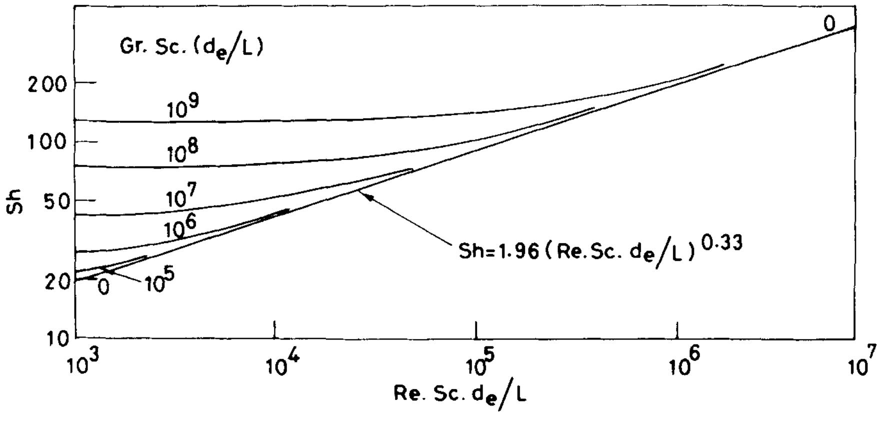

Figure 1. Variation of *Sh* with *Re* · *Sc* · d_e/L; effect of *Gr* · *Sc* · d_e/L.

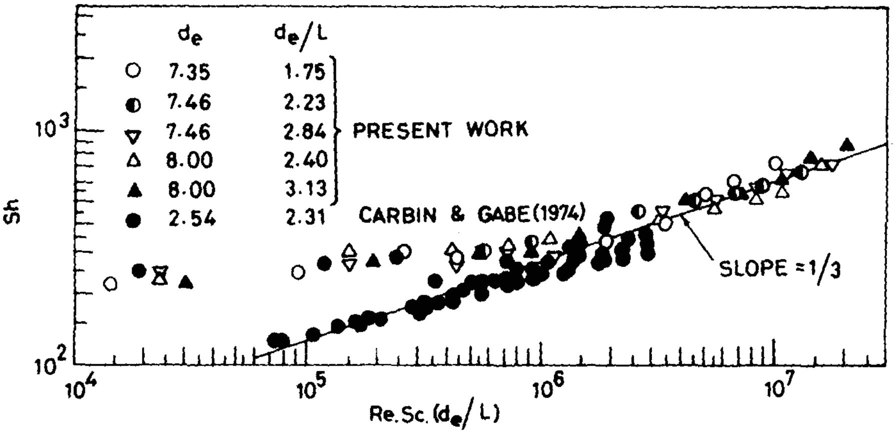

Figure 2. Variation of *Sh* with *Re* · *Sc* · d_e/L; experimental data.

or

$$Sh = 2.703[Re \cdot Sc \cdot \phi(a)(d_e/L)]^{1/3} \text{ (for } n = 1) \qquad (12)$$

which is similar to Eq. 6, developed empirically by Carbin and Gabe (1974). An analysis of the results of these workers, which is shown in Figure 4, indicates that Eq. 11 correlates the results quite satisfactorily (average deviation ±9%) and should be preferred to Eq. 5 (average deviation ±10%) due to its semitheoretical nature and lower average deviation.

Turbulent Regime

Since no data were available for mass transfer for developing turbulent flow in annulus, Pickett (1977) recommended the use of Eqs. 3 and 5 for the developing flow situations also.

It would be appropriate, however, to develop such correlations with the help of actual experimental data obtained with short transfer surfaces. Literature search revealed that data reported by Singh et al. (1971) are suitable for this purpose. The equations proposed by these workers are not based on the annular flow approach and have little relevance to the physical situation of the system. Their data are shown in Figure 5 as a $Sh[Sc(d_e/L) \cdot \phi(a)]^{-1/3}$ vs. Re plot. An analysis of the data revealed that for $Re < 6{,}000$, Eq. 3 can be used to correlate the data, provided proper allowance is made in the coefficient for the change in system geometry. Regression analysis indicated that

$$Sh = 0.305\, Re^{2/3} \cdot Sc^{1/3}(d_e/L)^{1/3}[\phi(a)]^{1/3} \qquad (13)$$

correlates the data with least deviation. This equation is no longer applicable to $Re > 6{,}000$ and a modified form of Eq. 5 would be more appropriate. From the analysis of the data in this regime, it was observed that

$$Sh = 0.0796\, Re^{0.8} \cdot Sc^{1/3} \qquad (14)$$

correlated the data quite satisfactorily. This analysis also revealed that, as expected by theory, (d_e/L) was no longer a controlling parameter in this regime.

NOTATION

a	= aspect ratio, D_i/D_o
d_e	= equivalent diameter, L
D	= molecular diffusion coefficient, L^2T^{-1}
D_i	= diameter of inner cylinder, L
D_o	= diameter of outer cylinder, L
g	= acceleration due to gravity, LT^{-2}
Gr	= Grashof number for Newtonian fluid, $gL_c^3\rho^2(\Delta\rho/\rho)/\mu^2$
Gr_p	= Grashof number for power law fluid, $gL_c^3\rho^2(\Delta\rho/\rho)/K'(8U_m/d_e)^{n-1})^2$
K	= power law consistency index, $ML^{-1}T^{n-2}$

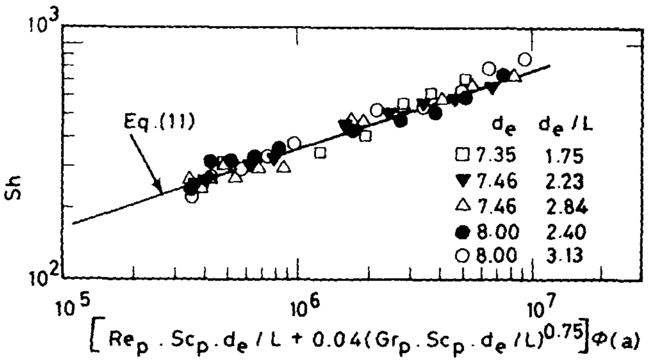

Figure 3. Correlation for non-Newtonian fluids; present experimental data (1% aq. CMC solution; n = 0.9365).

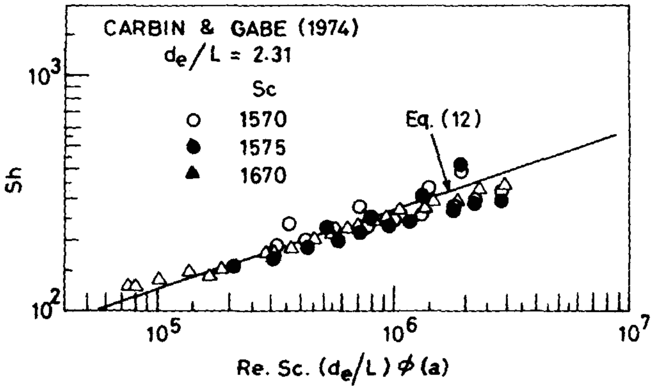

Figure 4. Correlation for Newtonian fluids; electrochemical mass transfer data.

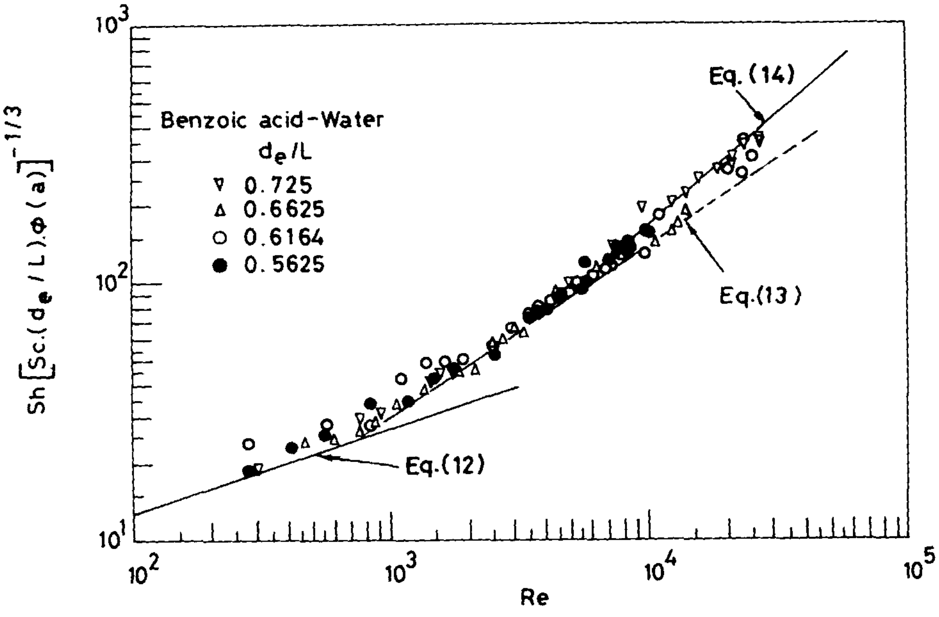

Figure 5. $Sh[Sc(d_e/L)\phi(a)]^{-1/3}$ **vs.** Re **plot; dissolution data of Singh et al. (1971).**

K' = generalized power law consistency index, $ML^{-1}T^{n-2}$
k_c = mass transfer coefficient, LT^{-1}
L = length of inner cylinder, L
L_c = characteristic length parameter, L
n = flow behavior index
Re = Reynolds number for Newtonian fluid, $d_eU_m\rho/\mu$
Re_p = Reynolds number for power law fluid, $d_e^nU_m^{2-n}\rho/K'8^{n-1}$
r = radial distance, L
Sc = Schmidt number for Newtonian fluid, $\mu/\rho D$
Sc_p = Schmidt number for power law fluid, $K'(8U_m/d_e)^{n-1}/\rho D$
Sh = Sherwood number, k_cd_e/D
U_m = mean flow velocity, LT^{-1}
μ = velocity, LT^{-1}

Greek Letters

$\Gamma(a,n)$ = function defined by Eq. 8
$\lambda(a,n)$ = dimensionless radius for maximum velocity
μ = viscosity, $ML^{-1}T^{-1}$
ν = kinematic viscosity, L^2T^{-1}
ρ = density, ML^{-3}
$\Delta\rho$ = density difference, ML^{-3}
τ = shear stress, $ML^{-1}T^{-2}$
$\phi(a)$ = function defined by Eq. 2
$\Omega(a,n)$ = dimensionless flow rate

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